

Summary of PhD Thesis: Full Stokes Ice Models and Subglacial Heat Sources

Numerical Simulations of Volcano-Ice Interaction

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Contents

1	Introduction	1
2	Research objectives	1
3	IceTools: a Full Stokes Finite Element Model for Glaciers	3
4	Numerical studies of ice flow over subglacial geothermal heat sources at Grímsvötn, Iceland, using Full Stokes equations	5
5	Progressive cooling of the hyaloclastite ridge at Gjálp, Iceland, 1996 - 2005	8
	Bibliography	11

1 Introduction

The doctoral thesis presented on May 25th, 2007 (Jarosch, 2007a) is based on the following papers, which are included as chapters in the thesis. A summary of the main results of each paper is given in subsequent sections.

- **Jarosch, A. H.:** *IceTools: a Full Stokes Finite Element Model for Glaciers*, Computers & Geosciences, 2007. (in review) **Section 3.**
- **Jarosch, A. H.,** Gudmundsson M. T.: *Numerical studies of ice flow over subglacial geothermal heat sources at Grímsvötn, Iceland, using Full Stokes equations*, Journal of Geophysical Research, Earth Surface, 2007, 112, F02008, doi:10.1029/2006JF000540 **Section 4.**
- **Jarosch, A. H.,** Gudmundsson, M. T., Högnadóttir, Þ, Axelsson, G.: *Progressive cooling of the hyaloclastite ridge at Gjálp, Iceland, 1996 - 2005*, Journal of Volcanology and Geothermal Research, 2007. (submitted) **Section 5.**

The primary aim of the work presented is to gain deeper understanding of the interaction between subglacial heat sources and ice, generally termed volcano-ice interaction. Common types of heat sources encountered are subglacial geothermal systems of different sizes as well as volcanoes or volcanic systems buried underneath glaciers or ice sheets. Previously reported sites featuring subglacial heat sources include several ice-filled calderas around the world (e.g. Clarke et al., 1989; Major and Newhall, 1989) and the large ice caps in Iceland, especially Vatnajökull and Mýrdalsjökull (e.g. Björnsson, 1988; Björnsson and Gudmundsson, 1993; Gudmundsson et al., 1997, 2002a, 2004, 2007; Jarosch and Gudmundsson, 2007). There is also evidence of an active volcanic area underneath the West Antarctic Ice Sheet, near the Whitmore mountains (Blankenship et al., 1993; Behrendt et al., 1994, 1995). Moreover, volcanic regions at high and middle latitude were ice covered during the Pleistocene and earlier glaciations (e.g. Velichko et al., 1997; Hickson, 2000).

2 Research objectives

By focusing on the effects of geothermal heat sources on ice dynamics in temperate glaciers, where ice temperatures are at the pressure melting point, the following research objectives are investigated:

- (i) Accurate numerical simulation of ice dynamics above subglacial heat sources with a state of the art ice flow model.
- (ii) Development of methods to infer subglacial heat source parameters by using glacial surface data: e.g. ice surface velocities, surface mass balance records and surface depressions volume changes.
- (iii) Reconstruction of the heat output record from the subglacial hyaloclastite ridge (edifice) formed in the 1996 Gjálp eruption, Vatnajökull, Iceland.

Earlier work on the interaction between subglacial heat sources and ice has not utilized numerical ice models to investigate quantitatively the ice dynamics involved. Until now, published research has used detailed calorimetric work to study heat output from subglacial geothermal areas and volcanoes, notably at Grímsvötn and Gjálp (Fig. 1) (Björnsson, 1988; Björnsson and

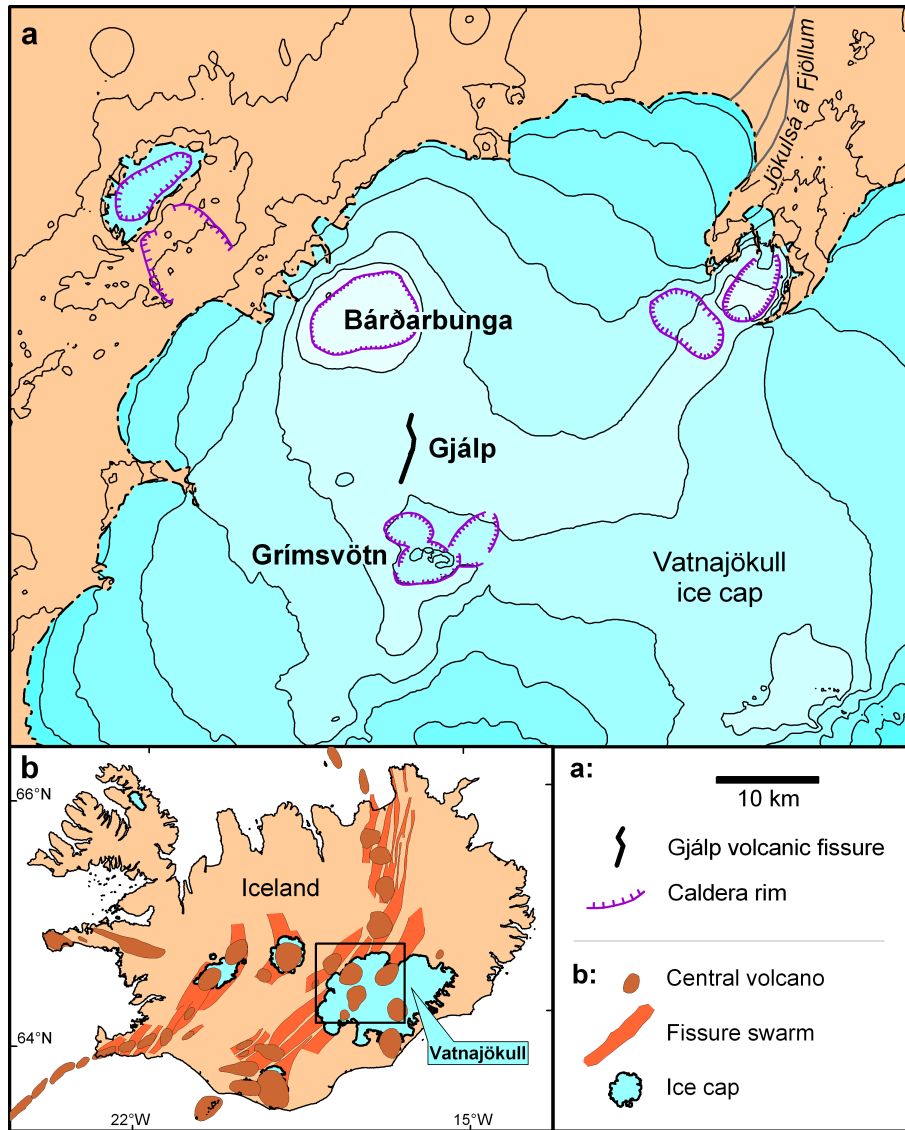


Figure 1: Location map. (a) A detailed map of the Vatnajökull ice cap, the Gjalp volcanic fissure and the Grímsvötn volcanic system. (b) An overview of the volcanic systems of Iceland.

Einarsson, 1990; Björnsson and Gudmundsson, 1993; Björnsson et al., 2001). In recent years, considerable advances have been made in ice flow modeling, both on the scale of ice sheets / ice caps and on a more local scale like small parts within a glacier. These new insights have made the use of a modern ice flow model to study volcano-ice interaction more feasible. A better understanding of the processes involved in such an interaction has a very practical dimension. As heat is transferred from the subglacial heat source to the ice, meltwater is created, which can cause jökulhlaups and lahars. These generally catastrophic events pose major threats in some volcanic regions, including Iceland (e.g. Major and Newhall, 1989; Björnsson, 2003; Gudmundsson, 2005).

The ice-covered volcanic regions in Iceland offer excellent opportunities to study the interaction of glaciers with subglacial heat sources at various scales. Subglacial eruptions are more common than elsewhere on Earth (e.g. Larsen, 2002; Gudmundsson, 2005; Thordarson and Larsen, 2007) and Iceland has many subglacial geothermal areas of different sizes and intensities (e.g. Björnsson, 1988; Gudmundsson et al., 2007). The western part of Vatnajökull hosts the highly active Grímsvötn central volcano and the recent eruption site of Gjalp. This area offers a unique variety of settings ranging from small ice surface depressions due to minor subglacial geothermal activity to large volcanic eruptions. Field data from this area collected over the last 10 years of intense volcanic and geothermal activity forms an important part of this study.

3 Icetools: a Full Stokes Finite Element Model for Glaciers

Detailed studies of glacier flow and deformation of ice require high resolution numerical modeling. The model presented in this paper, icetools, solves the Stokes equation including all components of the stress tensor, termed full Stokes, with the finite element method to enable such detailed studies (Jarosch, 2007b). Icetools is capable of running in parallel on computational clusters providing the computing power for large scale simulations.

Two different numerical tests were performed to demonstrate the capability of the model; (1) a comparison with the analytical solution for gravity driven plane flow down an inclined plane, and (2) flow over a Gaussian-shaped bed disturbance in comparison with the analytical transfer functions for this case. The second test involves time evolution of the surface geometry from an initially uniformly sloping surface. A linear rheology medium with a viscosity of $8 \times 10^{13} \text{Pa s}$ and a non-linear medium with Glen rheology exponent $n = 3$ and rate factor of $A = 23 \times 10^{-16} \text{s}^{-1} \text{kPa}^{-3}$ were studied. Both correlate well with the analytical solution and reach steady state, defined as when the maximum vertical movement drops below 5×10^{-5} times the mean ice thickness per year. The steady state was reached after 282 years for the linear case and 135 years for the non-linear case. The results of this comparison scaled to the mean ice thickness $h = 200 \text{ m}$ and for bedrock slope angle of 3° are displayed in Fig. 2. The main results of this paper can be summarized in the following conclusions:

- Numerical benchmark tests revealed a maximum error of the model in comparison with the analytical solution for a gravity driven plane flow down an inclined plane as low as 0.08 % for the smallest investigated grid size of 12.5 m on a squared grid, using the analytical solution as inflow and outflow boundary conditions. The different error behavior between squared and triangular grid elements was demonstrated as well for this test case. By investigating the flow over a single, Gaussian-shaped bed disturbance, it was possible to achieve correlation coefficients of 0.977 for a non-linear medium with $n = 3$ and 0.983 for a linear medium in comparison with the analytical transfer functions for this problem (Gudmundsson, 2003).
- The presented model, icetools, is a useful tool for teaching Full Stokes numerical ice models utilizing its simple-to-understand Matlab[®] interface. It was also successfully used to

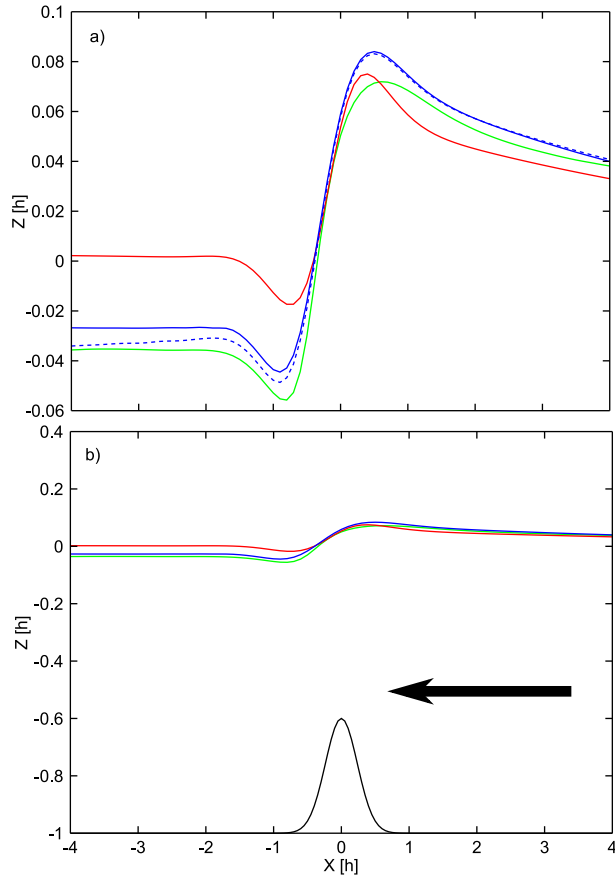


Figure 2: Comparison of three numerical model results with the analytical transfer functions for a linear medium (red). The numerical result for a linear medium with a viscosity of $8 \times 10^{13} \text{Pa s}$ (dashed blue), and a non-linear medium with $n = 3$ and $A = 23 \times 10^{-16} \text{s}^{-1} \text{kPa}^{-3}$ (green) are displayed for the 20h long model domain. The linear medium case for a 60h long model domain (blue) is also shown. All numerical solutions are plotted after they fulfilled the steady state criteria (cp. text). Main flow direction is indicated with the black arrow. (a) displays a close-up view of the surface response to the bed disturbance, whereas (b) includes the actual bed disturbance and the 20h linear medium case is not plotted here (Jarosch, 2007b).

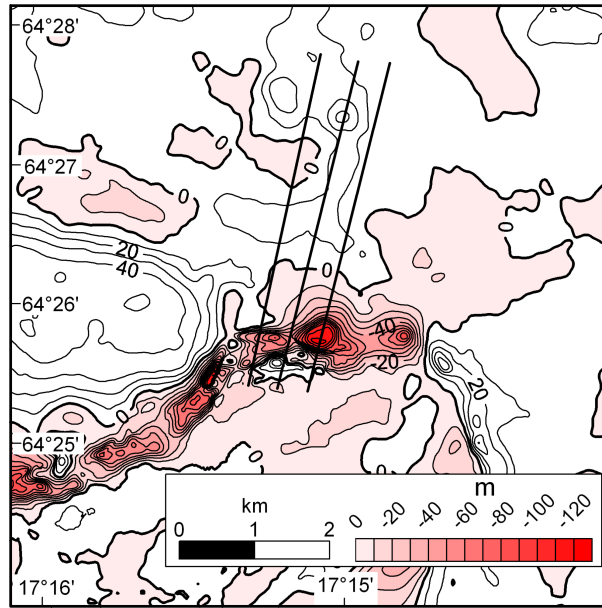


Figure 3: Contour map of the surface elevation change in the northeastern part of Grímsvötn between 2004 and 1998. Black lines mark the three profiles and negative elevation change is displayed in colors. The large uplift on the left reflects a rise in the level of the subglacial Grímsvötn lake (Jarosch and Gudmundsson, 2007).

investigate formation of ice surface depressions at the subglacial Grímsvötn volcano in Iceland (Jarosch and Gudmundsson, 2007).

4 Numerical studies of ice flow over subglacial geothermal heat sources at Grímsvötn, Iceland, using Full Stokes equations

Jarosch and Gudmundsson (2007) studied the evolution of a surface depression which formed between 1998 and 2004 on the eastern side of Grímsvötn (Figs. 1 and 3). The numerical ice flow model Icetools (Jarosch, 2007b) has been used to study ice dynamics above the geothermal heat source located on the caldera rim. Three north-south trending profiles (Fig. 3) were combined to create a numerical model of the region including a 400 m long geothermal heat source. The heat flux at the bedrock-ice interface was assumed to be distributed with the intensity being Gaussian-shaped over the 400 m long section.

To infer heat flux values at the bottom of the glacier, the numerical model has been used to simulate several scenarios of glacier surface evolution above the geothermal heat source based on different model parameters. The heat flux values implemented in the model and the stiffness of the ice (represented by the rate factor in Glen's flow law (Glen, 1955; Nye, 1957)) are the key model parameters to tune the simulated surface evolution to the measured surface evolution between 1998 and 2004. The stiffness parameter for the ice around Grímsvötn has been estimated by Aðalgeirsdóttir et al. (2000) and is used in this study as the favorable value. As soon as the

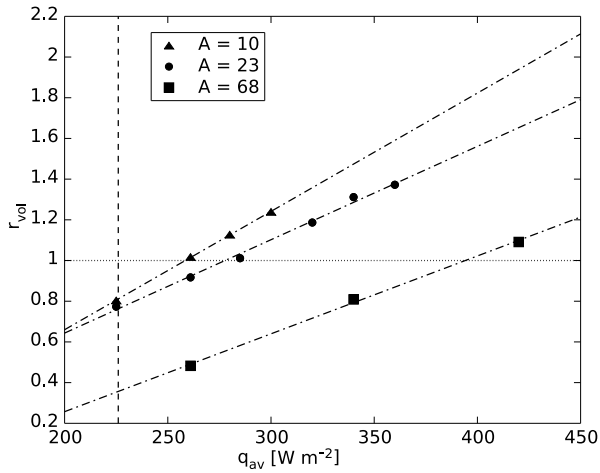


Figure 4: The ratio between the simulated and measured volume change (r_{vol}) during the model period from 1998 to 2004 as a function of heat flux (q_{av}) for 3 different rate factors, A ($\times 10^{-16} \text{s}^{-1} \text{kPa}^{-3}$). The dotted line marks the perfect volume fit, $r_{vol} = 1$ and the vertical dashed line marks the heat flux value q_{start} , estimated by the measured volume change, including a surface mass balance but ignoring all ice flux. Dash-dotted lines denote the linear regressions through the corresponding data sets.

modeled surface evolution matched the measured one, the model reproduces the behavior of the system sufficiently well and the implemented heat flux values can be used as an estimate for the heat flux values of the real geothermal heat source. A quality parameter is used to quantify how well the model fits the measured surface evolution. It is based on the surface depression volumes and has been termed r_{vol} . If $r_{vol} = 1$ a perfect match is achieved, values > 1 represent overestimation in volume by the model and values < 1 underestimation respectively.

Extensive numerical simulations of the system have revealed a strong dependency of inferred heat fluxes on the rate factor A in Glen’s flow law. As mentioned above, this rate factor describes the stiffness of the ice. It depends on several physical parameters, e.g. temperature, impurities and water content. More details about this model parameter can be found in Jarosch and Gudmundsson (2007) or Paterson (2001). A sensitivity study for three values of A has been carried out and the results are displayed in Fig. 4.

As can be seen in Fig. 4, the estimation of heat flux values depends heavily on the rate factor (stiffness) used. Heat flux values range from 260 - 390 W m^{-2} depending on the chosen ice stiffness. This dependency of the heat flux results can be explained by the ice dynamics above the geothermal heat source. The numerical simulations of the system have revealed the importance of horizontal ice movement into the surface depression as the depression is formed over time. Fig. 5b demonstrates the increase of horizontal ice flow into the depression over time. Before the depression was formed, horizontal ice velocities have been of the order of 10 m year^{-1} (not shown here) but at the end of the simulation horizontal ice velocities have increased to $\sim 70 \text{ m year}^{-1}$.

This sevenfold increase in ice velocities causes transport of more and more ice into the depression over time and the excess of ice mass influences the surface evolution of the model, which is the key to estimate heat flux values. The strong dependency on the ice stiffness is due to

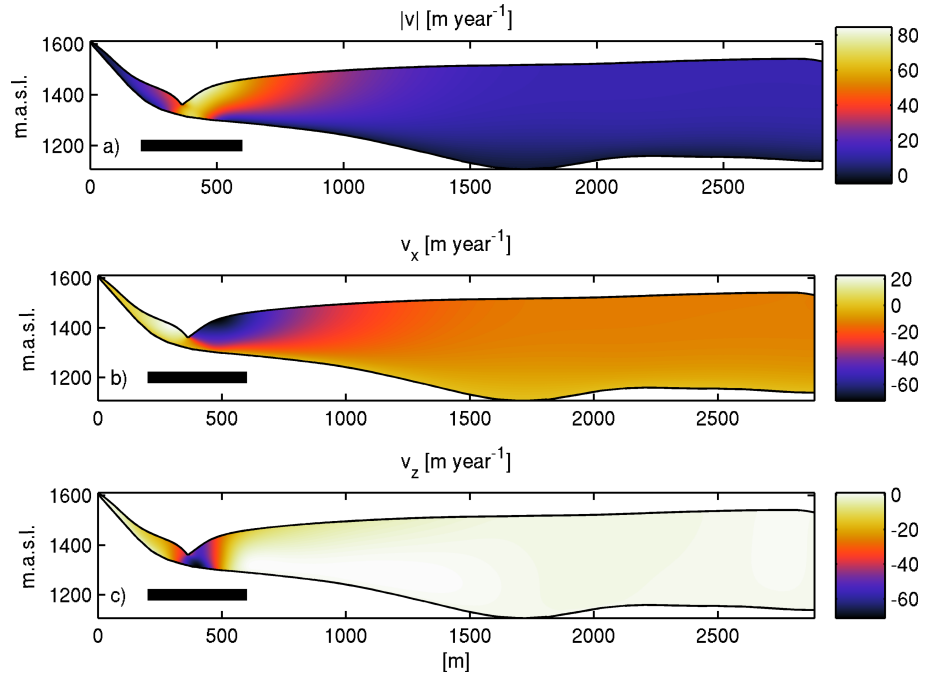


Figure 5: The final velocity flow field at the end of the simulation ($A = 23 \times 10^{-16} \text{s}^{-1} \text{kPa}^{-3}$). (a) the norm of the velocity vector, (b) the horizontal and (c) the vertical component as colors in m year^{-1} . The black bar indicates the extent of the heat source.

the fact that the softer the ice is (higher values of A) the more ice mass is transported into the depression. Since the stiffness of the ice is known for the Grímsvötn area, heat flux values can accurately be obtained with this method and the main findings of this work can be summarized as follows:

- A systematic 15-75 % underestimate of the geothermal flux can occur if inflow of ice into the depression is ignored. In this study, the amount by which the geothermal flux is underestimated is shown to be linearly related to the chosen values of the rate factor A in Glen's flow law. It is found that omission of the heat contribution used to melt inflowing ice would lead to an underestimate of 23 % if $A = 23 \times 10^{-16} \text{s}^{-1} \text{kPa}^{-3}$ is used.
- The average heat flux under the depression is estimated as 260-390 W m^{-2} , with 280 W m^{-2} corresponding to the favored value of A . These heat flux values are of the same order of magnitude as those at other, powerful subglacial geothermal areas in Iceland. The mean heat output of the 2.5-km-long depression over the survey period was 250-300 MW.
- The maximum deformation rate determined by the model is $1 \times 10^{-7} \text{s}^{-1}$, indicating that Glen's flow law (and thus the full Stokes approach used here) should be widely applicable to many cases where geothermal areas occur beneath ice.

5 Progressive cooling of the hyaloclastite ridge at Gjálp, Iceland, 1996 - 2005

The 1996 Gjálp eruption within Vatnajökull (Fig. 1) played an important role in the process of understanding subglacial eruptions. It was the first significant eruption within a large ice cap to be monitored in any detail. A considerable amount of new insights on processes occurring in subglacial eruptions under temperate glaciers, gained from the Gjálp observations, has already been published ((e.g. Björnsson et al., 2001; Gudmundsson et al., 1997, 2002a,b, 2004)). The reconstruction of the heat output record of the Gjálp edifice between 1996 and 2005 represents a major step forward towards understanding the formation and preservation of hyaloclastite ridges within glaciers.

Tuyas (table mountains), hyaloclastite ridges and sheets are common types of morphologies formed during subglacial eruptions (e.g. Mathews, 1947; Jones, 1969; Gudmundsson, 2005). During the Pleistocene, subglacial volcanic activity was a major, land-shaping process in Iceland, creating hyaloclastite ridges and tuyas. Those formations still dominate large parts of the present-day landscape. Subglacial volcanism turns out to be a major land forming process in other parts of the world as well, e.g. in western Canada (e.g. Mathews, 1947; Hickson, 2000) or on the Antarctic Peninsula, where extensive hyaloclastite regions are found (Smellie, 1999).

A comparison of the Gjálp edifice and its evolution with ridges formed during the Pleistocene under an ice sheet gives rise to several important questions. They can be formulated according to Gudmundsson et al. (2002a) as: (1) How well can the freshly formed Gjálp ridge, an initially unconsolidated pile of volcanic glass and tephra, withstand erosion from moving ice? (2) Can palagonitization prevent erosion and what is the rate of alteration for the Gjálp ridge? (3) What role does diversion of ice flow play in the preservation of the edifice? (4) Which general conclusions about the behavior of subglacial as well as subaqueous eruptions can be made?

A pile of initially unconsolidated volcanic material at the base of a glacier, subjected to fast ice flow is expected to suffer heavy erosion. Therefore large parts of a volcanic edifice may be removed over a relatively short period of time. It has been suggested that such erosion processes have occurred in West Antarctica (Behrendt et al., 1995) and may have happened also

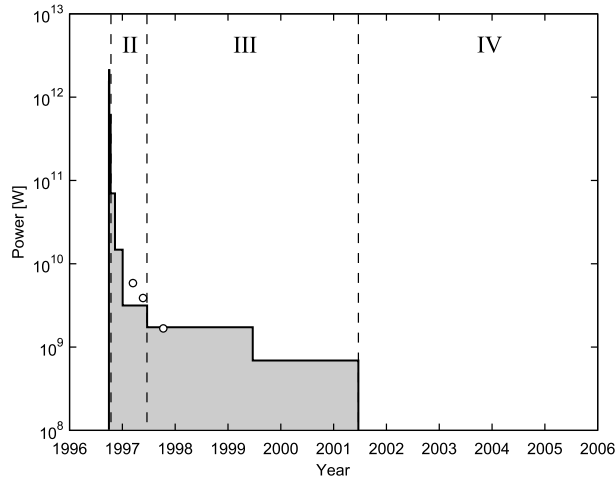


Figure 6: Evolution of the heat output of the Gjálp edifice 1996-2006. After June 2001 the power drops to $0 \pm 2 \times 10^8$ W. For comparison the results based on combined InSAR and GPS from Gudmundsson et al. (2002b) are marked with black circles. (II) marks the period from the end of eruption until June 1997, (III) June 1997-June (2001), and (IV) the period since June 2001 (see text).

in Iceland (Bourgeois et al., 1998). The Pleistocene ridges and tuyas found in Iceland are made of pillow lava, breccia and hyaloclastite, with hyaloclastite being the major component in some of the ridges (Schopka et al., 2006; Jakobsson, 1979). A common feature of these formations is that the volcanic glass has altered into palagonite, turning the loose pile of volcanic glass into consolidated rock (Jones, 1969; Jakobsson, 1979). This consolidation has been a key factor in preserving the edifices by making them resistant to glacier erosion. However, the rate at which this alteration occurs in the subglacial environment is unknown.

Another important aspect of studying the heat output record of the Gjálp edifice is to assess the potential for jökulhlaups to occur after the eruption. During the monitored period between late 1996 and mid 2005, no significant meltwater accumulation was detected at the Gjálp eruption site because the meltwater produced drained continuously into the Grímsvötn subglacial lake. Large jökulhlaups are highly likely to occur during or right after an subglacial eruption. The likelihood of occurrence decreases rapidly after an eruption unless an unusual bedrock geometry favors water accumulation within the glacier.

To be able to give answers to the questions stated above, the heat output record of Gjálp between 1996-2005 has been reconstructed using an extensive data set of field measurements, e.g. glacier surface topography maps, ice surface velocity measurements, and accumulation measurements. This data set has already been published (Gudmundsson and Högnadóttir, 2003; Jarosch et al., 2005; Jarosch, 2007a) and has been combined with a numerical glacier flow model of the inflow area north of Gjálp using the Icetools software (Jarosch, 2007b). The combined data can be used to estimate heat output from the Gjálp edifice over time (details of the method are given in Jarosch et al. (2007)) and the result is shown in Fig. 6.

In the attempt to explain the cooling history of Gjálp, a liquid-phase buoyancy-driven heat transport model has been used in combination with estimated average temperatures within the edifice (Jarosch et al., 2007). The heat transport model compares likely permeability values for the edifice with the actual heat flux record and also includes a simple, heat conduction based

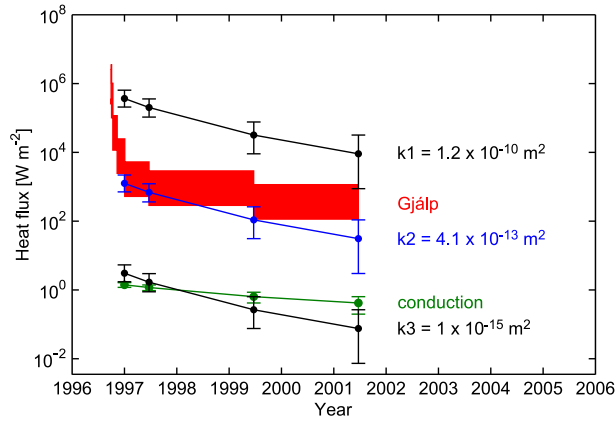


Figure 7: Heat flux estimates based on three different permeability values, estimated temperature conditions and assuming liquid phase buoyancy driven flow. The Gjálp heat flux record is displayed in red. For comparison, heat flux values assuming conduction and the same temperature conditions are shown in green. A best fit can be obtained using permeability values of $1\text{-}2 \times 10^{-12} \text{ m}^2$.

model for comparison (Fig. 7).

Three permeability values were used to estimate possible heat flux values, two from Surtsey and one from Hawaii. For unaltered hyaloclastite in Surtsey $k_1 = 1.2 \times 10^{-10} \text{ m}^2$ was used and for altered hyaloclastite at a depth of 60-100 m b.s.l. $k_2 = 4.1 \times 10^{-13} \text{ m}^2$ (Stefansson et al., 1985). The very old hyaloclastite from Hawaii, which was found at a depth of 2000-3000 m b.s.l. has a permeability of $k_3 = 1 \times 10^{-15} \text{ m}^2$ (Dannowski, 2002). As Fig. 7 clearly indicates, likely permeability values for the Gjálp edifice are similar to those found in altered hyaloclastite at Surtsey. This result in combination with the other findings from this study (Jarosch et al., 2007) lead to the following main conclusions:

- The heat output history of Gjálp can be divided into four episodes: (I) The eruption (13 days), (II) end of eruption until June 1997, (III) June 1997 - June (2001), and (IV) the period since June 2001. During episode (I) heat output dropped from initially $> 2 \times 10^{12} \text{ W}$ to $7 \times 10^{10} \text{ W}$ and further decreased to $3 \times 10^9 \text{ W}$ by the end of episode (II). An average value of $1.2 \times 10^9 \text{ W}$ characterized episode (III) and no significant heat output was measured in episode (IV).
- The total eruptive energy was $1.50 \pm 0.28 \times 10^{18} \text{ J}$, estimated from the volume of erupted material. A remarkable $\sim 64 \%$ of the total energy was released during the eruption itself and by June 2005, only some 5% remained within the edifice.
- The heat remaining in the edifice at several points in time has been determined and its average temperature estimated. The temperature dropped from $\sim 240^\circ\text{C}$ at the end of the eruption to $\sim 128^\circ\text{C}$ after three months and $\sim 107^\circ\text{C}$ after nine months. In mid 1999 an average temperature of $\sim 58^\circ\text{C}$ is estimated and $\sim 38^\circ$ by mid 2001, with little cooling occurring since.
- Using a liquid-phase buoyancy-driven convection model and the derived edifice temperatures, it is found that the cooling history is consistent with permeability values of order

10^{-12} - 10^{-13} m², similar to that estimated for consolidated hyaloclastite in the island of Surtsey, but inconsistent with a pile of loose tephra. This may indicate that the edifice consolidated to dense hyaloclastite in the first 1 or 2 years. However, this remains speculative.

- No traversing ice flow over the edifice was observed in the surface velocity data record, indicating that the surface depression closure still dominates the local ice flow field.

Bibliography

- Aðalgeirsdóttir, G., Gudmundsson, G. H., Björnsson, H., 2000. The response of a glacier to a surface disturbance: a case study on Vatnajökull ice cap, Iceland. *Annals of Glaciology* 31, 104–110.
- Behrendt, J. C., Blankenship, D. D., Damaske, D., Cooper, A. K., 1995. Glacial removal of late Cenozoic subglacially emplaced volcanic edifices by the West Antarctic ice sheet. *Geology* 23, 1111–1114.
- Behrendt, J. C., Blankenship, D. D., Finn, C. A., Bell, R. E., Sweeney, R. E., Hodge, S. R., Brozena, J. M., 1994. Evidence for late Cenozoic flood basalts(?) in the West Antarctic rift system revealed by the CASERTZ aeromagnetic survey. *Geology* 22, 527–530.
- Björnsson, H., 1988. Hydrology of Ice Caps in Volcanic Regions. Rit XLV. Societas Scientiarum Islandica, Reykjavík.
- Björnsson, H., 2003. Subglacial lakes and jökulhlaups in Iceland. *Global and Planetary Change* 35, 255 – 271.
- Björnsson, H., Einarsson, P., 1990. Volcanoes beneath Vatnajökull, Iceland: evidence from radio-echo sounding, earthquakes and jökulhlaups. *Jökull* 40, 147–168.
- Björnsson, H., Gudmundsson, M. T., 1993. Variations in the thermal output of the Subglacial Grímsvötn Caldera, Iceland. *Geophysical Research Letters* 20 (19), 2127 – 2130.
- Björnsson, H., Rott, H., Gudmundsson, S., Fischer, A., Siegel, A., Gudmundsson, M. T., 2001. Glacier-volcano interaction deduced by SAR interferometry. *Journal of Glaciology* 47 (156), 58 – 70.
- Blankenship, D. D., Bell, R. E., Hodge, S. M., Brozena, J. M., Behrendt, J. C., Finn, C. A., 1993. Active volcanism beneath the West Antarctic ice sheet and implications for ice-sheet stability. *Nature* 361, 526 – 529.
- Bourgeois, O., Dauteuil, O., van Vliet-Lanoë, B., 1998. Pleistocene subglacial volcanism in Iceland: tectonic implications. *Earth and Planetary Science Letters* 164, 165–178.
- Clarke, G. K. C., Cross, G. M., Benson, C. S., 1989. Radar Imaging of Glaciovolcanic Stratigraphy, Mount Wrangell Caldera, Alaska - Interpretation Model and Results. *Journal of Geophysical Research - Solid Earth and Planets* 94, 7237 – 7249.
- Dannowski, G., 2002. Untersuchungen des thermo-hydraulischen Feldes und der Wärmeflussdichte in der Nähe eines aktiven Mantelplumes (Insel Hawaii) mit Hilfe von Bohrlochtemperaturmessungen. Str02/03, Technische Universität Berlin.

- Glen, J. W., 1955. The creep of polycrystalline ice. *Proceedings of the Royal Society, London* A228 (1175), 519 – 538.
- Gudmundsson, G. H., 2003. Transmission of basal variability to a glacier surface. *Journal of Geophysical Research* 108 (B5), 2253, doi:10.1029/2002JB002107.
- Gudmundsson, M. T., 2005. Subglacial volcanic activity in Iceland. In: Caseldine, C., Russel, A., Hardardóttir, J., Knudsen, O. (Eds.), *Iceland - Modern Processes and Past Environments*. Elsevier, pp. 127 – 151.
- Gudmundsson, M. T., Högnadóttir, Þ., 2003. Gjálp 1997- 2002: Maelingar á Ísskridi og varmaafli. Tech. Rep. RH-02-2003, Raunvísindastofnun Háskólans.
- Gudmundsson, M. T., Högnadóttir, Þ., Kristinsson, A. B., Gudbjörnsson, S., 2007. Geothermal activity in the subglacial Katla caldera, Iceland, 1999-2005, studied with radar altimetry. *Annals of Glaciology* 45, 66–72.
- Gudmundsson, M. T., Pálsson, F., Björnsson, H., Högnadóttir, Þ., 2002a. The hyaloclastite ridge formed in the subglacial 1996 eruption in Gjálp, Vatnajökull, Iceland: present day shape and future preservation. In: Smellie, J. L., Chapman, J. L. (Eds.), *Volcano-Ice Interaction on Earth and Mars*. Vol. 202. Geological Society, London, Special Publications, pp. 319 – 335.
- Gudmundsson, M. T., Sigmundsson, F., Björnsson, H., 1997. Ice-volcano interaction of the 1996 Gjálp subglacial eruption, Vatnajökull, Iceland. *Nature* 389, 954 – 957.
- Gudmundsson, M. T., Sigmundsson, F., Björnsson, H., Högnadóttir, Þ., 2004. The 1996 eruption at Gjálp, Vatnajökull ice cap, Iceland: efficiency of heat transfer, ice deformation and subglacial water pressure. *Bulletin of Volcanology* 66, 46 – 65.
- Gudmundsson, S., Gudmundsson, M. T., Björnsson, H., Sigmundsson, F., Rott, H., Carstensen, J. M., 2002b. Three-dimensional glacier surface motion maps at Gjálp eruption site, Iceland, inferred from combining InSAR and other ice-displacement data. *Annals of Glaciology* 34, 315 – 322.
- Hickson, C. J., 2000. Physical control and resulting morphologic forms of Quaternary ice-contact volcanos in western Canada. *Geomorphology* 32, 239 – 261.
- Jakobsson, S. P., 1979. Outline of the petrology of Iceland. *Jökull* 29, 57 – 73.
- Jarosch, A. H., 2007a. Full stokes ice models and subglacial heat sources - numerical simulations of volcano-ice interaction. Ph.D. thesis, University of Iceland.
- Jarosch, A. H., 2007b. Ictools: a Full Stokes finite element model for glaciers. *Computers & Geosciences* (in review).
- Jarosch, A. H., Gudmundsson, M. T., 2007. Numerical studies of ice flow over subglacial geothermal heat sources at Grímsvötn, Iceland, using Full Stokes equations. *Journal of Geophysical Research - Earth Surface* 112, F02008, doi:10.1029/2006JF000540.
- Jarosch, A. H., Gudmundsson, M. T., Högnadóttir, Þ., 2005. Gjálp 2003-2005: Depression development, ice flux and heat output. Tech. Rep. RH-20-2005, Jarðvísindastofnun Háskólans.
- Jarosch, A. H., Gudmundsson, M. T., Högnadóttir, Þ., Axelsson, G., 2007. Progressive cooling of the hyaloclastite ridge at Gjálp, Iceland, 1996 - 2005. *Journal of Volcanology and Geothermal Research* (submitted).

- Jones, J. G., 1969. Intraglacial volcanoes of the Laugarvatn region, south-west Iceland - I. Quarterly Journal of the Geological Society London 124, 197 – 211.
- Larsen, G., 2002. A brief overview of eruptions from ice-covered and ice-capped volcanic systems in Iceland during the past 11 centuries: frequency, periodicity and implications. In: Smellie, J. L., Chapman, M. C. (Eds.), *Volcano-Ice Interaction on Earth and Mars*. Geological Society, London, Special Publications, 202, pp. 81 – 90.
- Major, J. J., Newhall, C. G., 1989. Snow and Ice Perturbation during Historical Volcanic-Eruptions and the Formation of Lahars and Floods - A Global Review. *Bulletin of Volcanology* 52, 1 – 27.
- Mathews, W. H., 1947. "Tuyas", flat topped volcanos in northern British Columbia. *American Journal of Science* 245, 560 – 570.
- Nye, J. F., 1957. The distribution of stress and velocity in glaciers and ice sheets. *Proceedings of the Royal Society London* 239, 113 – 133.
- Paterson, W. S. B., 2001. *The Physics of Glaciers*, 3rd Edition. Butterworth Heinemann, Oxford.
- Schopka, H. H., Gudmundsson, M. T., Tuffen, H., 2006. The formation of Helgafell, southwest Iceland, a monogenetic subglacial hyaloclastite ridge: Sedimentology, hydrology and volcano-ice interaction. *Journal of Volcanology and Geothermal Research* 152, 359 – 377.
- Smellie, J. L., 1999. Lithostratigraphy of Miocene-Recent, alkaline volcanic fields in the Antarctic Peninsula and eastern Ellsworth Land. *Antarctic Science* 11, 347 – 363.
- Stefansson, V., Axelsson, G., Sigurdsson, O., Gudmundsson, G., Steingrimsson, B., 1985. Thermal conditions of Surtsey. *Journal of Geodynamics* 4, 91–106.
- Thordarson, T., Larsen, G., 2007. Volcanism in Iceland in historical time: Volcano types, eruption styles and eruptive history. *Journal of Geodynamics* 43, 118–152.
- Velichko, A. A., Kononov, Y. M., Faustova, M. A., 1997. The last glaciation on Earth: Size and volume of ice-sheets. *Quaternary International* 41/41, 43–51.